Volume 13 (2013)

Interface: The Journal of Education, Community and Values

3-18-2013

Directed Reading: A strategy for teaching students to read mathematics

Nancy Ann Neudauer
Pacific University

Directed Reading: A strategy for teaching students to read mathematics

Directed Reading: A Strategy for Teaching Students to Read Mathematics

by Nancy Ann Neudauer, Ph.D.
Department of Mathematics & Computer Science, Pacific University

Abstract- Getting students to read the textbook is one of the major hurdles I face in teaching undergraduate mathematics. Students who do read the new material before hearing a lecture on it tell me that it is a tremendous help in understanding the lecture. Yet so few students do this.

Directed Reading offers students little choice but to read the book before coming to class. The students like it because it guides them through the reading and they follow the lecture more easily. Moreover, it saves precious class time on routine problems, and we can concentrate on subtleties and intricacies in the material.

1. Learning to Read

I have a problem getting my students to read their mathematics book. For years I would tell them to read the section before the lecture on that section. Occasionally a student would tell me that he or she had done this and that "it really helped." But for the most part the students admitted that they had not. I thought that maybe they did not wantto read the book, or maybe they did not like to read it. Many of them were struggling with mathematics. If reading the book really helped, why wouldn't they do it? Then it dawned on me—they didn't know how to read a mathematics book.

I thought back to how I had learned to read. Not back to sounding out the words and reading "See spot run. See Jane watch spot run," but rather to how I had learned to read for content. I remembered a long time ago—in second grade or fourth grade—reading a chapter in a social studies book and at the same time filling out a worksheet. The worksheet would have lines such as:

_____ is the capital of Spain.
The major exports of Spain are _____ and _____.
Most of the people of Spain speak _____.

As I read, I would pick out pieces of information and fill in the blanks. At this point I was reading more than just words; I was expected to get some information out of what I was reading. The worksheet helped guide me, or

direct me, in my reading.

2. Learning to read mathematics

Maybe I was right about the capital of Spain idea. But it seemed that 12 years after filling in Madrid we should be capable of a bit more than filling in blanks. And perhaps even more importantly, this was not exactly the type of content I wanted students to read for in a mathematics class. It wasn't that the students could not read, but that they could not read mathematics.

What exactly dowe want the students to glean from this reading? When they read a section before the lecture, whatwill help them to understand the lecture?

I was confronted with one more problem that helped me answer those questions: my lectures were each 65 minutes. I had been accustomed to teaching 50 minute lectures, and had even then thought the students' attention spans for new material were 35 or 40 minutes at best. Now with a 65 minute lecture I was worried I would be wasting too much of the time without the full attention of the class.

I could almost watch the students drift off. If at the board lecturing in calculus, for example, I would first introduce a new idea. The students would perk up a bit as I did some very simple examples. I would proceed to do some more complicated examples and would start to lose their attention. And just as I got to the subtleties—the really good stuff that maybe they really did need me for and could not just get from the book—they would drift off completely. I thought that if I could just get to that subtle stuff earlier—in the first 30 minutes—then maybe I could hold their attention long enough.

So this was my goal—to get to the more difficult, subtle problems in the first 30 minutes of lecture. I began to think about which material the students could learn from the book on their own. Students had complained from time to time that I didn't assign problems that were like the examples in the section. From precisely here came my idea: I would assign problems that were exactly like the examples, but I would assign them before I lectured on that material. That, I realized, was the sort of reading that the students could do, and in fact that they craved. Maybe at first they would not be reading all the words—but they would be absorbing some of the content. If I had lectured on section 2.3, an assignment might look like this:

Section 2.3 # 8, 10, 14, 15, 19, 22, 35, 36, 41

Section 2.4 # 1, 3, 4

where the Section 2.4 problems are mimics of the example problems in the text. I have had the opportunity to use Directed Reading in both upper- and lower-level courses, in courses for both mathematics majors and non-majors.

The following are a few examples of how I have used it.

3. Directed Reading in Calculus

Directed Reading in Calculus proved to be much more effective than I had anticipated. Where I had done several elementary examples in the past, I decided I would cut back to one, then move on to more complex ones. One day, for example, I was introducing integration by parts. I wrote ∫xcosx dx on the board. A student spoke up and said, "We've already done that." I pointed out that they had done , and he said, "It's the same thing." I then suggested that we try ∫tx-tdt. Another student pointed out that they had done ∫we-xdx. A third student said, "Can't we work on some harder ones, since that's what we'll have to do in the homework and on the test?"

As a result, I find that I can save 20 minutes of lecture time, the students have all at least attempted the more basic problems on their own and at their own pace, and they are able to follow the lecture better as we discuss the subtleties. And the students have what they want—homework problems that are exactly like the examples in the text. Occasionally, such as after an exam, I will not include a directed reading portion on the next assignment. The students will ask for it, arguing, "How can they be expected to follow the lecture if they haven't even looked at the material."

The types of problems that work well as directed reading problems are either a homework problem that closely mimics an example in the text or problems the students have done before but may need to review. With the former, the students carefully work through the example problem. With the latter, students are able to review at their own pace. For instance, in calculus I have found that students often do not remember very well absolute values, logarithms, solving systems of linear equations, and inequalities. Yet when I have spent a little time reviewing these in class, the students who remember them feel like I am wasting class time, while the students who do not remember think that I am going too fast. I find a pace that pleases no student. In working these problems before the lecture, the students are able to choose an appropriate speed until they feel comfortable with this review material.

Directed Reading serves another role here: In addition to saving lecture time, Directed Reading helps to bridge a gap between students of different abilities and with different backgrounds. Having students work problems before class, whether new problems or review problems, gets them all on the same page at the beginning of the lecture. Further, it allows the students to get there at their own pace.

4. Directed Reading in Abstract Algebra

Directed Reading in Abstract Algebra seemed to be a more challenging endeavor at first than it was in calculus. I was struck by the amount of material I needed to cover in this course, so I thought about what sorts of problems were more routine. Primarily I wanted to grade the students' proofs, but I also realized that to do the proofs the students would need to understand the structure as well. I found many straightforward and computational problems at the beginning of each section that required manipulating group elements, multiplying permutations, constructing group tables, etc.

I assigned these problems before lecturing on each section. Instead of collecting these computational problems, I would select some at the beginning of the lecture. Each student would write on a piece of paper which of those he or she could do. I would call on students to put their solution on the board. The students would receive one point for each problem they said they could do, but would lose 20 points if unable to do a problem when called on. The students received immediate feedback on their work instead of waiting for it to be graded, and they also had answers to their questions before the lecture had begun. I thought the negative points would keep the students honest and that the students would only claim to be able to do problems they actually could do, but students had different confidence in their answers, and some are risk takers. If I find this is not working well with a particular class, I change to giving points for students who volunteer to put a problem on the board. I teach abstract algebra differently from how I was taught it, and I have learned from Fraleigh's article "Happy Abstract Algebra Classes." [4]

In comparison with previous times I had taught abstract algebra, there were noticeable differences. I used to work through many specific examples, covering every detail. Now I am able to cover deeper topics. Instead of presenting a lot of examples, I can mention them and the students will recall them as they have more familiarity with them due to working this all out on their own.

5. Implementation Difficulties

I have found the major stumbling block in the implementation of Directed Reading is the collection of the daily assignments and the quick turnaround necessary to incorporate the responses in that day's lecture. Turning in problems at the beginning of class does not allow me time to read them and modify the class to address questions raised or to skip material that is already clear to the students. Since the students are turning in a Directed Reading assignment on each class day, it becomes a paper-management and grading nightmare. Even if you can return the assignment at the next class, this rate of feedback

may be inadequate for this method. We can circumvent the class preparation issue by having students e-mail the professor the day before; this allows time to modify the lecture to meet students' questions and understanding. This helps with lecture preparation but does not provide the quick feedback to the student that some of the newer online mathematics systems would. It also works better when the answers are in words rather than equations or mathematical notation. Some books have reading questions built in, like Beezer's First Course in Linear Algebra. [2] Others have computational problems that align well with examples in the section. A web forum can encourage online discussion of the topics before the class period, but this might not perfectly fit this model. With the huge number of new internet and online resources available, I believe that some combination of them will provide an appropriate forum to collect, assess, give feedback, and employ student understanding of new material. Ideally, the correct combination of these resources will help students to read better for content, thus not only providing the instructor and student feedback, but helping to develop their mathematical reading skills.

Many of the references on teaching students to read mathematics are targeted toward K-12 teachers [1] or pre-date internet current resources [3, 7], and are not widely employed. Recently, a nice overview called How to Read Mathematics appeared that will provide a great introduction to students (or faculty!) using Directed Reading in a course. [5]

Last year, to try to address some of the implementation and feedback issues with Directed Reading, I used WeBWorK for the Directed Reading questions. What is WeBWork? According to the Mathematical Association of America (MAA) website,

> *WeBWorK is a well-tested homework system for delivering individualized problems over the web. By providing students with immediate feedback as to the correctness of their answers, students can be encouraged to make multiple attempts until they succeed. With individualized problem sets, students can work together but will have to enter their own work to receive credit. The instructors are provided with real-time statistics, resulting in lesson plans that can be customized to better serve students. [6]*

We have a WeBWorK installation on our server, so students are given local accounts. WeBWorK gives them immediate feedback on their response, and instructors can choose to give them more than one chance to get a correct answer.

There were several advantages to WebWork. Students did receive immediate feedback to the Directed Reading questions and could then try again,

potentially immediately correcting misconceptions. Students did not need to try to fit mathematical notation into an e-mail—some questions were multiple choice, but even for ones that were not, students commented that WeBWorK was easier for them than other online homework systems they had used. There were also some limitations. For example, problems from the existing library did not always align well to the book we were using. Instructors can write their own problems, but this can be a lot of work, and even modifying existing questions does not always go smoothly. Moeover, some of the existing questions had unexpected answers. Finally, I was not able to find suitable questions for each section or lecture.

I had planned to only use WeBWorK for Directed Reading questions, but quite unexpectedly I started using it for sets of review questions between exams and to check student understanding of routine computational exercises. Students liked these questions; if they had a good understanding of the material, it was an instant reward, and if they did not, they could practice and try again immediately. Also, these problems tended to be shorter and more straightforward than the text's homework problems. The increase of student understanding was measurable from this practice. However, there was still frustration and resistance. Glitches in the system and unexpected answers frustrated both students and myself. Some students felt like this was an additional requirement on top of their regular homework. When I implement this again, I will be sure to reduce the amount of other work commensurate with the time spent on WeBWorK assignments.

6. Other student reactions

Another obstacle in employing Directed Reading was a notion the students had that it was unfair to expect them to do a problem before I had "taught" them how to do it. Pointing out that they might have to read Romeo and Juliet before it was discussed in class, or to read a philosophy paper or a psychology chapter, did not shed any light. In fact, when I gave a talk about directed reading, I got the same reaction from some of the mathematics faculty in the audience. They asked how I could expect the students to do a problem that I had not yet discussed.

To combat this, the second semester I employed Directed Reading, I talked with the students about it in class, explaining what I thought the benefits were, and asking for their feedback. In subsequent semesters I have also put in the syllabus the following:

Read each section before I discuss it in class. Once we have gone over a section, reread it, and work through the examples. Sometimes it is difficult to read a mathematics book before coming to class. To guide you in this, each

homework assignment will have a Directed Reading component. A few problems from the coming section with constitute this portion of the assignment. The problems will not be complex, but instead will mimic the examples in the text or will follow directly from the reading.

I have a Directed Reading component to each assignment. Now that the students expect it, their reactions are primarily positive, but it still remains a challenge in some classes. Students who like it feel that they are better prepared for the lecture, and often point out that the problems from the upcoming section are easy. And they are, in comparison with the challenging problems in the section I have lectured on, which are not just like the examples in the text.

There are benefits to using Directed Reading in both upper- and lower-level courses, in courses for both mathematics majors and non-majors. Non-majors benefit because they develop their mathematical reading skills, which they can bring to other technical reading they might have in future classes, and in their careers. The hope is that this will make them stronger independent learners, which is the transformation of students that we really hope to see in the undergraduate experience. Since we expect mathematics majors to read difficult and advanced mathematics independently for their capstone projects, developing these skills early and throughout their mathematics courses can better prepare them. Research mathematics presents unique difficulties for undergraduate students. It is unlike other sciences in that students cannot be put in a lab working on a hands-on experiment as part of a larger effort. It demands instead that they work conceptually with a high degree of independence on an abstract, unsolved problem. Even if a student simply wants to read some articles on new mathematics, finding an article an undergraduate can read is difficult. Once an appropriate article is found, having experience reading mathematics will be an asset. I hope that in learning to read mathematics in the earlier years of college, students will be more prepared to take on their own research projects in their later years.

Notes

[1] Barton, M. L., Heidema, C. (2002). *Teaching reading in mathematics: A supplement to Teaching Reading in the Content Areas: If Not Me, Then Who? 2nd Edition* (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.

[2] Beezer, R.A. (n.d.). A First Course in Linear Algebra. Retrieved from: http://linear.pugetsound.edu

[3] Borasi, R. & Siegel, M. (1990). Reading to learn mathematics: New connections, new questions, new challenges. *For the Learning of Mathematics*, 10, 9-16.

[4] Fraleigh, J.B. (2001). Happy Abstract Algebra Classes. *MAA Focus*, 21(8), p. 13.

[5] Gouvea, F. & Simonson, S. , (2011). How to Read Mathematics. In S. Simonson, R*ediscovering Mathematics* (pp. xvii-xxix). Mathematical Association of America.

[6] WeBWorK (n.d.). Mathematical Association of America. Retrieved from http://webwork.maa.org/wiki/

[7] Raphael, T. E. & Gavelek, J. R., (1988). Question-related activities and their relationship to reading comprehension: Some instructional implications. In G. Duffy & L. Roehler (Eds.), *Comprehension instruction: Perspectives and suggestions* (pp. 234-250). New York: Longman.