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S5, God and Numbers

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It has become somewhat ontologically fashionable for theists to embrace anti-realist views of abstract objects.¹ On pain of consistency, however, I will argue that any theist who accepts the S5 ontological argument for the existence of God should also accept a parallel S5 ontological argument for the existence of abstract objects. This is because the same ontological and modal inferences thought to demonstrate the existence of God can also be used to demonstrate the existence of numbers. I further argue that being ontologically consistent here might come at the price of being theologically unacceptable, as the argument for numbers would seem to vindicate a platonistic view of *abstracta*—but platonism is incompatible with the traditional concept of God. I conclude by suggesting what I see as the only way out of this dilemma, which is to embrace a view akin Augustine’s where *abstracta* exist “nowhere but in the very mind of the Creator.” Something like the Augustinian view of *abstracta*, therefore, can be seen as an indirect theological consequence of the S5 ontological argument for the existence of God.

1. The S5 Ontological Argument

Alvin Plantinga has argued that on the basis of the S5 axiom $\diamond \Box P \rightarrow \Box P$ (if it is possible that necessarily P, then necessarily P) and an important qualification (mentioned below), a sound ontological argument for God’s existence can be constructed.² Plantinga’s argument begins by pointing out what might appear to many to be an obvious truth about the traditional theistic conception of God; namely, that

(N1) Necessarily, if God exists, then necessarily God exists

which is to say that, by definition, God is the kind of being such if he exists at all, he necessarily exists. Call this *the necessity intuition*. Operating under the modal domain of metaphysical necessity (broadly logical necessity), a simple version of the argument can then proceed as follows:

- (1) It is possible that God exists
- (2) If it is possible that God exists, then it is necessary that God exists
- (3) It is necessary that God exists

Granted that God is the sort of being such that were he to exist, he would necessarily exist, it follows that if it is possible that God exists, God does exist. The crucial step in the argument is the possibility premise, (1). “But are there any possible beings—that is, merely possible beings, beings that don't in fact exist?” Plantinga asks. “If so, what sorts of things are they? Do they have properties? How are we to think of them? What is their status? And what reasons are there for supposing that there are any such peculiar items at all?”³ Plantinga avoids these knotty problems in a stroke by speaking of *properties* and the worlds in which they are or are not instantiated or exemplified rather than of possible *beings* and the worlds in which they do or do not exist. This move places the argument among what Peter van Inwagen calls “manifestly valid” ontological arguments. Manifestly valid ontological arguments “proceed from a premise that asserts of a set of properties that it satisfies certain conditions, to the conclusion that there exists something that exemplifies that set of properties,”⁴ where these properties are such that if exemplified or instantiated at all, are exemplified or instantiated necessarily (*n*-properties for short). Keeping in step with the above argument, the necessity intuition here is

- (N2) Necessarily, if *n*-property is instantiated, then necessarily *n*-property is instantiated

from which one can argue that the possible instantiation of some *n*-property will entail the necessary existence of the object or being that has that property.

What might be God's *n*-properties? For the argument to establish the traditional concept of God, such properties would include minimally omnipotence, omniscience, and omnibenevolence. Plantinga collectively refers to the having of this trio of *n*-properties as *maximal excellence*. Further, a being is said to have the property of *maximal greatness* if it has *maximal excellence* necessarily, or in all possible worlds. Replacing the being ‘God’ in the first syllogism with the property *maximal greatness*, the argument establishes a being that exemplifies the property of *maximal greatness*—God. A more refined version will read:

- (1') It is possible that *maximal greatness* is instantiated
- (2') If it is possible that *maximal greatness* is instantiated, then it is necessary that *maximal greatness* is instantiated
- (3') It is necessary that *maximal greatness* is instantiated

but it remains the case that the meaning of premises (1') through (3') is synonymous with what is more directly asserted by premises (1) through (3), respectively.

2. A Parallel Argument

Now it seems we can construct a parallel argument for certain abstract objects, such as numbers.⁵ Mathematical concepts are often understood to operate within the modal domain of metaphysical necessity. This is because numbers seem to stand in logically necessary relations. For example, consider the proposition ‘ $2+2=4$.’ It seems impossible to conceive how, upon understanding the concepts of 2 and 4, adding 2 to 2 would not equal 4. In no possible world where we have the same concepts of the terms in the proposition ‘ $2+2=4$ ’ could the proposition possibly be false. If it is hard to see how the proposition ‘ $2+2=4$ ’ could possibly be false, it is perhaps even harder to see how the proposition could be nonexistent. If metaphysical necessity governs mathematical concepts, and mathematical objects actually exist, then these objects must exist of metaphysical necessity. So if there were any other beings to which the necessity intuition applies, it would be to abstract, mathematical objects like numbers. Thus it is relatively uncontroversial that

(N3) Necessarily, if numbers exist, then necessarily numbers exist

Replacing God with numbers, the same modal inferences used in the above argument can be used in the following argument:

- (4) It is possible that numbers exist
- (5) If it is possible that numbers exist, then it is necessary that numbers exist
- (6) It is necessary that numbers exist

But before the parallel is pronounced exact, the same qualifications made in the argument for God should be made here. We can accommodate the possible-beings-to-properties move by either thinking of numbers themselves as kinds of *n*-properties or that there are *n*-properties of numbers. This is no mere philosophical chicanery; such views have enjoyed prominence in the philosophy of mathematics since the analyses of Mill, Frege, Cantor, and Russell.⁶ A contemporary defender of the former option is David Armstrong, who considers numbers as properties of aggregates.⁷ Some machine, for example, can have the property of being 10-parted, or perhaps a square, 4-sided. Similarly, Penelope Maddy thinks that numbers are properties of sets in the same way *length* and *weight* are properties of physical objects.⁸ Sets have number properties, just as physical objects have dimensional properties. Charles Lambros takes a neo-Fregean approach and argues that number words in sentences function as predicates of objects.⁹ So when one observes three marbles, the property of *three-ness* is predicated of the marbles. The predicates do not necessarily have to refer to physical objects. They could, like on Maddy’s view, refer to abstract objects like sets and their members.

Representing the latter option are contemporary philosophers such as John Bigelow and Peter van Inwagen. Rather than considering numbers themselves as properties of aggregates, as Armstrong does, Bigelow suggests that numbers are *constituents* of such properties.¹⁰ So the 10-parted machine has the property *having 10 parts* just in case there are things which number 10, each of which is a part of the machine. Peter van Inwagen forcefully argues that there are necessarily existent mathematical properties, the having of which could be by mathematical objects like numbers.¹¹ Van Inwagen asks, “Is there any set of properties such that, necessarily, there exists at least one object that has every member of that set? Or, as we shall say, is there any set of properties that is necessarily instantiated? Well, yes,” he concludes, citing mathematical properties that would satisfy the necessity intuition (*m*-properties for short). Van Inwagen defines an *m*-property as “an essential property that entails the property of being a mathematical object,” and gives the examples of *being prime*, *being a set of reals*, *being everywhere continuous and nowhere differentiable*, and *being-the-set-of-real-numbers*.¹²

Though I will proceed with van Inwagen’s view, whichever view one finds most attractive is not likely to make much of a difference so long as the view amounts to the *possible* existence of *m*-properties. Thus, it would seem we have a parallel argument that succeeds precisely where the many popular parodies of the ontological argument do not: a possible being the concept of which satisfies the necessity intuition in the same way that the concept of God does. Whereas unicorns, flying spaghetti monsters, and desert islands are not the sort of *beings* that have *n*-properties or the sort of beings about which the necessity intuition is true, God and numbers uniquely are. *Mutatis mutandis*,

- (4') It is possible that *m*-properties are instantiated
- (5') If it is possible that *m*-properties are instantiated, then it is necessary that *m*-properties are instantiated
- (6') It is necessary that *m*-properties are instantiated

again bearing in mind that the meaning of premises (4') through (6') is synonymous with what is more directly asserted by premises (4) through (6), respectively. Indeed, the argument for numbers might even be considered stronger than the argument for God because the former’s possibility premises, (4) and (4'), are likely to enjoy *more* intuitive support among philosophers than the latter’s (1) and (1'). This is because the concept of number is simple and unanalyzable, unlike the concept of God.

3. Problems with the Parallel

It is important to stress that I am making no judgment as to the soundness of these arguments here. My thesis is hypothetical: *if* a theist thinks the S5 ontological argument

for God is sound, *then* he should also think the argument for numbers is sound, or at least it is hard to see on what grounds he could accept the former and not the latter.

It is also important to note that semantic nominalisms are irrelevant to the truth of (4). This is because it remains the case that on such theories it is *possible* that numbers exist, or that *m*-properties are possibly instantiated.¹³ Analogously, one would not challenge the possibility premise in the argument for God by appealing to verificationism because, even if true, it is not a consequence of the view that God does not in fact exist. Surely it is possible that God exists or *maximal greatness* is possibly instantiated even though talk of such is meaningless. To truly dispense with the existence of God, a much stronger, ontologically anti-realist view is needed, such as logical positivism. And for numbers, a view such as fictionalism would do the trick.

It will be recalled that if numbers exist, they necessarily exist, which is to say that if numbers do not exist their existence is impossible. The correct way to challenge (4) therefore would be to take up the incredibly difficult task of showing that it is logically impossible that numbers exist. I suppose one could do this by demonstrating incoherence in the concept of numbers. But given that mathematical expression is almost universally recognized as the hallmark of logical consistency, such a demonstration is farcical. It is hard even to conceive how such a demonstration would proceed. We at least can conceive how this could be done in the case of God; *viz.*, by demonstrating the impossibility of the attributes that make up *maximal excellence*. But how could one demonstrate the incoherence of a simple and unanalyzable concept like that of a number? Even Hartry Field, an avid mathematical anti-realist, maintains that although mathematical entities do not exist, they could have; that is, their existence is not logically impossible.¹⁴

A more promising objection to the modal argument for numbers comes from Colin Cheyne in his masterful polemic against platonism, *Knowledge, Cause, and Abstract Objects*. At the heart of Cheyne's objection is the necessity intuition for numbers:

(N3) Necessarily, if numbers exist, then necessarily numbers exist

Cheyne concedes that if (N3) is true then it is a short argument in S5 to (6). He writes, "the notion of [logical necessity] allows us to argue from the logical possibility of numbers to their actual existence." Cheyne seems to be sensitive to the strength of (N3), so "the solution," he says, "is not so much in the outright rejection of (N3), but in its modification, or perhaps in a more careful reading of it." How does Cheyne suggest we do this? He continues, "We can retain (N3) so long as the necessity in the consequent is a more restricted necessity than logical necessity."¹⁵ By adopting this strategy, one could expose the parallel arguments as resting on equivocal notions of necessity.

The problem, however, is that Cheyne offers no account of a restricted form of mathematical necessity. Instead, he simply waves the issue by saying “the precise nature of the necessity of numbers need not be fully explicated here.”¹⁶ But if this objection is to succeed, we not only need such an account, we need additional reasons for preferring it over the modality in which mathematics is traditionally understood. Preferring some other kind of modality over metaphysical necessity without argument is bound to appear either *ad hoc* or qualify as textbook special pleading. Even Field notes that “if you think that talk of ‘metaphysical necessity’ makes sense,” as any theist who accepts the S5 ontological argument for God should, then “you can even regard it as part of the concept of God or of numbers that anything falling under those concepts is metaphysically necessary; if so, then it will be conceptually necessary that if there are numbers then they are ‘metaphysically necessary beings’, and analogously for God.”¹⁷

Field, however, doesn’t think talk of metaphysical necessity makes sense, and so picks up where Cheyne leaves off by developing an alternative account of mathematical necessity. Field thinks that what we are really intuiting in modal assertions like (N3) is something he calls *conservativeness*, not metaphysical necessity.¹⁸ The idea of conservativeness in mathematics is roughly *internal consistency* (it is conservative because it is thought to be consistent only insofar as it accurately reflects how the physical world works). “Truths” within conservative theories are not necessary, just consistent explanatory utilities (hence the scare quotes). To illustrate, suppose some proposition p is implied by conservative theory T . Now suppose a subsequently developed conservative theory T^* implies $\sim p$ or renders p superfluous. If T^* is thought to better represent the way the physical world is, so much for the ‘truth’ of p . According to Field, propositions like ‘ $2+2=4$ ’ are like p in T . And the reason we cannot help but think p is necessary is because we cannot conceive of how it could be that $\sim p$ in T . But Field’s point is that T itself need not be true. Field speculates, “perhaps many realists would be content to say that all they meant when they called mathematical claims necessarily true was that they were true and that the totality of them constituted a conservative theory.”¹⁹ So perhaps a theistic critic of the argument for numbers, though accepting metaphysical necessity with respect to (N1), could retain Cheyne’s strategy and follow Field in his attenuated account of mathematical necessity with respect to (N3).

Although Field’s theory of conservativeness has been challenged on other grounds, the main difficulty in the present context is that it does not accomplish what it needs to. As Stephen Yablo explains, the intuition underlying the truth of propositions like (N3) is simply that, *as an individual proposition*, it must be true, not that it would be true in the context of a larger theory. “But the latter is what we should say if our intuition is really of conservativeness. For conservativeness is a property of particular statements only seen as exemplars of a surrounding theory.”²⁰ The fact remains that (N3) captures

exactly what our modal intuitions suggest. Therefore, Field proves to be of no aid to Cheyne's strategy.

Even if it is granted that no modal equivocation is going on, Cheyne is prepared to counter the argument for numbers with a parallel of his own. Cheyne thinks that if the S5 ontological argument for numbers is sound, "then so is [an] argument for the existence of number-excluders."²¹ Number-excluders, according to his parody, are "entities that exist necessarily, if they exist at all, and whose existence in a world excludes the existence of numbers in that world."²² The parody:

- (7) It is possible that number-excluders exist
- (8) If it is possible that number-excluders exist, then it is necessary that number-excluders exist
- (9) It is necessary that number-excluders exist

And because (6) and (9) cannot both be true, one of the arguments must be unsound. In order to give Cheyne a fair hearing, it would be congenial to recast his argument in terms of possible properties rather than entities or objects. The problem, however, is that I have not the faintest idea of what such properties would be. An *n*-property that excludes *m*-properties? Unless some coherence can be given to a being-to-properties move for (7), the argument need not detain us. This notwithstanding, one could make short work of the parody simply by denying (7). Cheyne tries to preempt this, conceding that while "there might be some queasiness concerning such weird entities as number-excluders," nevertheless "it is difficult to see on what grounds someone could insist that they are a logical impossibility whilst numbers are not."²³

But there is no difficulty here at all. Just ask yourself *how many* number-excluders exist. The answer, presumably, is at least *one*. But this entails that at least the number one necessarily exists, which further entails the existence of at least two number-excluders, which entails that at least the number two necessarily exists. One could repeat this *ad infinitum*. Even if we try to avoid this by postulating a number-excluder for the number of number-excluders, the problem repeats itself because there would be at least *one* ultimate number-excluder, and so on. What this shows is that the existence of numbers is conceptually or logically prior to the existence of number-excluders. And this suggests, ironically, that if such weird entities did exist, they would not be in the same modal domain as numbers! But assuming they are both metaphysically necessary, if (9) is true, it entails (6). But if (6) is true, then (7) is necessarily false. Therefore, (7) is logically impossible and the numbers argument remains intact.²⁴

Thus, the most promising strategies for undermining the S5 argument for numbers prove fruitless, leaving us with no good reason to think it does not truly parallel the argument for God.

4. Theological Problems with Platonism

The conclusion of the argument for numbers is not as metaphysically tidy as the argument for God. Whereas (3) is fairly straight-forward in what it asserts, that God exists, (6) would seem to usher in an infinite number of abstract objects; in other words, traditional platonism.²⁵ According to platonism, abstract objects exist *a se* independent of mind as part of the necessary framework of reality. But this understanding of *abstracta* is fundamentally at odds with the traditional concept of God. According to the traditional, Judeo-Christian concept of God, there are two kinds of reality: God and not God. The former is self-sufficient and necessarily existent, the latter is not. This distinction gives rise to a further distinction, that between Creator and creature. It is then said that all things that are not God, therefore, are part of creation. But these distinctions leave no room for abstract objects platonistically conceived. For example, it is hard to see how one could square the existence of platonic objects with the attribute of aseity and, as a consequence, the doctrine of creation out of nothing. As Paul Copan and William Lane Craig explain, “the...reason [platonism] is unacceptable to orthodox theists, is that Platonism is incompatible with the doctrine of *creatio ex nihilo* and so fundamentally compromises God’s aseity. For Platonism posits infinite realms of being that are metaphysically necessary and uncreated by God.”²⁶ This is a most vexing problem no matter how you look at it.²⁷

But suppose the theist finds himself irresistibly attracted to platonism, and so tries to avoid the aforementioned problem by tweaking the notion of creation from “all things not God” to “all *creatable* things not God.” Given that *abstracta* are not creatable, one could say *abstracta* do not properly fall under the scope of creation and so in not inconsistent with (at least) the doctrine of creation out of nothing. It seems to me that far from avoiding the problem this tweaked notion of creation makes matters worse. Consider the nature of modal reality in general as it relates to the nature of God. Most theists will likely agree that God’s nature acts as a delimiter of modal reality; that is, what is or is not possible will be determined by God’s nature. For example, that God could actualize a state of affairs inconsistent with his own nature seems impossible.²⁸ But this seems to suggest that there cannot be a modal reality independent of God, which is exactly what platonism entails. In fact, on platonism *abstracta* must *also* serve to delimit *possibilia*. For example, God couldn’t create a world where $2+2=5$ not because anything about God’s nature prevents such an incoherence, but because it is a brute and inexplicable fact, even for God, that $2+2=4$. *Abstracta* exist as a minimal blueprint for any creatable reality for God. Thus, God creates not strictly *ex nihilo*, but also according to *abstracta*. So even the tweaked notion of creation as ‘all things creatable by God’ gives us at best creation *ex abstracta*.

But even deeper problems begin to surface for the theist with platonist sympathies. Suppose we ask *why* modal reality is the way it is. If God must create according to *abstracta*, as we just saw, then it seems the platonist must insist that modal reality is the

way it is because *abstracta*, not God, is the very essence of uncreatable reality. *Abstracta* is the delimiter of modal reality, including God's nature, for presumably God's own nature must conform to a logical blueprint laid down by *abstracta*. Here, *abstracta* are explanatorily (if not causally) prior to God's nature. This attenuates God's aseity even more than it first seemed. But a further difficulty with this account is that the nature of certain *abstracta* clearly seem explanatorily posterior God's nature; e.g., necessarily true moral propositions such as 'It is wrong to torture babies for fun.' The traditional theist can explain why this is a necessarily true proposition—because moral goodness, as God's very nature, delimits what necessary moral truths there are, 'it is wrong to torture babies for fun' being among them. But here the platonist confronts a dilemma: he must either deny that the nature of all *abstracta* is explanatorily prior to God's nature, or he must reaffirm this but deny that the *abstracta* typically seen as explanatorily posterior God's nature, such as necessarily true moral propositions, are so.

The former move seems very arbitrary and *ad hoc*. It has us believe that only *certain abstracta* are the way they are independent of God. But what calls for the difference? It is hard to see what, if anything. If one says "God's nature," then incoherence looms, for really this amounts to saying "God's nature determines which *abstracta* are explanatorily posterior his nature." Not only is this suspiciously circular, it faces the further question: "What about God's nature determines which *abstracta* are explanatorily posterior his nature?" It is hard to see how the platonist could answer this without affirming that it is God's nature that delimits modal reality, including *abstracta*.

Either way the first horn of this dilemma does not seem palatable. The second horn states that all *abstracta*, even necessarily true moral propositions, are the way they are *just because*. They are the very essence of uncreatable reality, and they delimit all possible realities, including God's nature as it were. But this horn immediately puts the Euthyphro dilemma back on the table, for it can no longer be said that certain things are good because God's nature is such, but, quite literally, God's nature is such because certain things are good. This would also remove any potential for the moral argument for God's existence, as the explanation of any moral truth would not be in terms of God's nature, but in terms of a reality apart from God. Because it is not a brute fact that God's nature is good, the question of *why* God's nature is good for the platonist becomes very difficult to answer. It is just convenient that it is. Moreover, strictly speaking, God's *nature* cannot be good, for if it were, goodness would be an uncreated and essential aspect of Him and so would need no further explication. But if the platonist maintains that goodness is not to be identified strictly with God's essential nature, then God's goodness becomes a contingent matter—yet another consequence that would make most traditional theists uncomfortable. Therefore, it seems impossible to square platonism with the traditional concept of God.

Conclusion

What, then, is the theist who accepts the S5 ontological argument for God to do with the parallel argument for numbers? Perhaps a cue can be taken from Richard Swinburne. Upon reflecting on what is traditionally meant by naming God as ‘Creator of all things,’ Swinburne too notes that it should be “understood with the qualification ‘apart from himself’ or, more precisely, ‘apart from anything the existence of which is entailed by his own existence.’”²⁹ God’s nature and whatever it might entail would thereby fall outside the scope of creation. So understood, the theist could combine the conclusions of the two arguments not by tweaking traditional doctrine but by adopting something like Augustine’s view where *abstracta* are “thought to exist nowhere but in the very mind of the Creator.”³⁰ It is important to see that on this view, *abstracta* are not construed as being the *created products* of the divine intellect, but as uncreated divine ideas.³¹ As Augustine explains, “the ideas are certain original and principle forms of things, i.e., reasons, fixed and unchangeable, which are not themselves formed and, being thus eternal and existing always in the same state, are contained in the Divine Intelligence.”³² Augustine’s view thus avoids the many problems posed by platonism. For example, grounding *abstracta* in this way plausibly explains why modal reality is the way it is without recourse to anything but the essence of uncreated reality—God’s nature.³³ Now, question remains as to whether such a view is itself coherent and could enjoy independent argument. But as the only theological acceptable option for the theists who endorse the S5 argument for God, it is exactly this question that deserves further exploration.³⁴ Should such an Augustinian view turn out to be coherent, that there are parallel S5 ontological arguments for God and numbers would hardly be surprising to the theist who sees the locus of necessity *qua* divinity.

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¹ E.g., Richard Swinburne, William Lane Craig, Brian Leftow.

² There are of course others who have offered modal versions of the ontological argument, but I focus on Plantinga's because of its popularity. The most detailed presentation appears in *The Nature of Necessity* (New York: Oxford University Press, 1979), ch. 10. For the sake of clarity I have withheld the following details. The S5 ontological argument uses axiom B (*Brouwersche* axiom), which states that $P \supset \Box \Diamond P$, from which one can deduce $\Diamond \Box P \supset \Box P$. Operating under the modality of broadly-logical necessity, or metaphysical necessity, the formal structure of the necessity intuition is

$$(N) \quad \Box[(\exists x)Px \supset \Box(\exists x)Px] \qquad \text{(Definition)}$$

which should be assumed true of P in the following formal structure of the generic ontological argument outlined in the text:

$$\begin{array}{ll} (1) \quad \Diamond \Box(\exists x)Px \supset \Box(\exists x)Px & \text{(Axiom B, S5)} \\ (2) \quad \Diamond \Box(\exists x)Px & \text{(Premise)} \\ (3) \quad \Box(\exists x)Px & \text{(1 \& 2, MP)} \end{array}$$

from which of course it follows that $(\exists x)Px$ is true in the actual world. It should also be mentioned that (1) is qualified by something like Plantinga's Restricted Ontological Principle, which states that any world in which a singular predicative proposition is true, is one in which its subject *exists*, where '∃' can be understood in terms of instantiation. See *The Nature of Necessity* (New York: Oxford University Press, 1979): 150-152.

³ Plantinga 1974, 102.

⁴ van Inwagen 1977, 375.

⁵ It is possible to generalize the point to encompass other abstract objects. But mathematical objects, of which numbers are paradigmatic, are especially convenient examples.

⁶ Some of the historical accounts are surveyed in Christopher Menzel, “Theism, Platonism, and the Metaphysics of Mathematics,” *Faith and Philosophy* 4 (1987): 365- 382. See also George Bealer, *Quality and Concept* (New York: Oxford University Press, 1982), ch. 6.

⁷ Cited in Bigelow 1988, 38-39.

⁸ Maddy 1990, 86-98.

⁹ Lambros 1976.

¹⁰ Bigelow 1988, 34-44.

¹¹ van Inwagen 1977.

¹² *Ibid.*, 377, 379.

¹³ “Semantic nominalism is to be distinguished from metaphysical nominalism. Semantic nominalism does not need to deny the existence of universals and sets: it only denies that they are signified by general terms.” John Bigelow, “Semantic Nominalism,” *Australasian Journal of Philosophy* 59 (1981): 404.

¹⁴ Views that construe mathematical existence as contingent face the difficulty of explaining what numbers are contingent on, if not some other necessarily existent being (mathematical or other). This is Crispin Wright and Bob Hale’s principle criticism of Hartry Field’s contingent nominalism. See their “Nominalism and the Contingency of Abstract Objects,” *Journal of Philosophy* 89/3 (1992): 111-135. In his response article, Field makes the interesting comment: “... given that it is impossible to give an account of what the existence of mathematical entities could be contingent on, the necessary existence of such entities would follow...But I have no idea what such a sense of possibility could be; and it's hard to see how one could be found that wouldn't resuscitate Anselm's argument for God along with the formally analogous argument for numbers” (Field 1993, 293), which is of course the principle insight of this paper. Aside from this, the rest of Field’s comments on the ontological argument are unfortunately confused.

¹⁵ Cheyne 2001, 107-108. Where \Box is logical or metaphysical necessity, m_p are m -properties and \Box_m is some restricted form of mathematical necessity, Cheyne suggests that we could read (N3) as $\Box[(\exists x)m_px \supset \Box_m(\exists x)m_px]$, from which we would not get as ontologically dramatic a conclusion.

¹⁶ *Ibid.*, 108.

¹⁷ Field 1993, 286. Field also sees a resemblance between God and numbers in that just as there certain “core” analytic truths of necessity built into the concept of numbers, there are “core” analytic truths built into the anselmian concept of God. Contra Field, Neil Tennant argues against this analogy in “On the

Necessary Existence of Numbers,” *Noûs* 31/3 (1997): 307-336. Tennant writes, “Inquiring into the existence and necessity of numbers takes us into quite different logical terrain than inquiring into the existence and necessity of God” (307). While Tennant happily grants that there are “core” analytic truths of necessity about numbers, he says “God does not seem to have much interesting internal structure. He becomes interesting only when featuring as a posit in various hypothetical interactions with human destiny. And ... its claim to analyticity is undermined by the lack of other conceptual, rather than empirical, controls on the positing” (318-319). Frankly I am not able to make much sense out of Tennant’s remarks on this. As I see it, Tennant’s issue with Field’s analogy is that because the concept of God seems to rely on or be conditioned by human interest, it lacks the kind analyticity or “conceptual control” characteristic of the necessity that numbers have. Sure, there are differing concepts of God, but the analogy and argument here are not concerned with those concepts about which the necessity intuition is not true. And perhaps it is *necessity* Field had in mind as one of the “core” analytic truths entailed by the concept of divine perfection, though the appeal to the anselmianism is unclear. Nevertheless, it remains the case that the necessity intuition is and always has been true of the traditional Abrahamic concept of God, the so-called ‘God of the philosophers,’ and the anselmian concept.

¹⁸ On the different ways of interpreting mathematical necessity, see the interesting discussion between Hartry Field and Crispian Wright and Bob Hale. A good moderating voice is Mark Colyvan, “Conceptual Contingency and Abstract Existence,” *The Philosophical Quarterly*, 50/198 (2000): 87-91.

¹⁹ Field 1989, 241-242. Field writes, “if mathematics together with a body N of nominalistic assertions implied an assertion A which wasn’t a logical consequence of N alone, then the truth of the mathematical theory would hinge on the logically consistent body of assertions $N + \sim A$ not being true. But it would seem that it must be possible and/or not a priori false, that such a consistent body of assertions about concrete objects alone is true; if so, then the failure of conservativeness would show that mathematics couldn’t be ‘true in all possible worlds’ and/or a priori true.” See Field, *Science Without Numbers* (Oxford: Basil Blackwell, 1980), 13.

²⁰ Yablo 2002, 172.

²¹ Cheyne 2001, 106.

²² Ibid. I have changed the numbering on the propositions quoted from Cheyne.

²³ Ibid., 107.

²⁴ Cheyne also argues that a contradiction results once we “consider the possibility that numbers do not exist”:

(4*) Possibly, numbers do not exist $\diamond\sim(\exists x)Nx$

Now, the negation of (4*) is logically equivalent to (6):

(6[̄]) Not-possibly not: numbers exist $\sim\diamond\sim(\exists x)Nx$

the problem being that “(6), and hence (6[̄]), follows from (4) and (5). So, (4) and (4*) cannot both be

true, at least if (5) is true” (ibid). The contradiction Cheyne sees is between (4) and (4*), as he is clearly construing one to be the negation of the other when he says, “(4) and (4*) cannot both be true.” But here Cheyne has apparently confused the modal negation

(Q) $\diamond \sim P$ (possibly not-P)

which is representative of his (4*) with

(R) $\sim \diamond P$ (not-possibly P)

which is the proper way to negate (4). So instead of (4*), we should get

(4**) Not-possibly, numbers exist $\sim \diamond (\exists x)Nx$

which is to say it is *impossible* that numbers exist, not that they possibly do not. Because this objection could be (and has been) raised against *any* S5 ontological argument, I do not treat it as an objection to the parallel. After all, I am assuming the logic of the S5 ontological argument is valid in this paper.

²⁵ There are several ways one could demonstrate that platonism entails the existence of the actual infinite, but it is especially obvious with numbers. For example, any natural number n will always have a successor $n+1$ such that $n < n+1$, and $n+1 < n+1+1$, *ad infinitum*. Because any natural number can always be added to, it follows that there can be no finite limit to the natural numbers. Or, as Stuart Shapiro succinctly puts it, “It is an axiom of arithmetic that zero is a natural number and a theorem that for every natural number n , there is a number $m > n$ such that m is prime. Together these imply that there are infinitely many prime numbers.” Stuart Shapiro, *Thinking About Mathematics* (New York: Oxford University Press, 2000): 201. One recalls Euclid’s proof that from any finite set of primes one can deduce a further prime that lacks membership. Paulo Ribenboim catalogues dozens of ways the proof can be laid out. See his *The New Book of Prime Number Records* (New York: Springer-Verlag, 1995), Ch. 1. Moreover, if we consider zero as the first non-negative integer, the rest of the set of natural numbers follows in a different way. 0, as the first natural number, entails the second; namely, 1. But this entails the third: 0, 1, and 2. And so on. One could adduce non-arbitrary reasons for starting with zero. For example, the number zero properly numbers the amount of objects in a null-world and members in an empty set. Thus, Peter Fletcher, “The ‘actual’ view of infinity goes naturally with the general philosophy of realism. . . From such a standpoint it is natural to suppose that there is an infinite of objects.” Fletcher, “Infinity” in (ed.) Dale Jacquette, *Philosophy of Logic* (The Netherlands: Elsevier, 2007), 526-527. It is not without reason that terms like “plenitudinous” and “prolifigate” are often used to describe platonism’s object-commitments. In fact, platonist Mark Balaguer defends what he calls “full-blooded Platonism,” the view that all mathematical objects that could possibly exist actually do exist (which would be an infinite number). Furthermore, commitment to the existence of the actual infinite might run into other, purely philosophical problems the natural theologian might not find palatable, such as the impossibility of the existence of an actually infinite number of objects. Interestingly, if the argument for numbers is indeed parallel, then accepting the ontological argument might mean rejecting one of the main legs of the kalam cosmological argument. But J. P. Moreland doesn’t think so. See his “A Response to a Platonistic and to a Set-Theoretic Objection to the Kalam Cosmological Argument,” *Religious Studies* 39 (2003): 373–390.

²⁶ Copan and Craig 2004, 173

²⁷ Alvin Plantinga explores these issues in *Does God Have a Nature?* (Milwaukee: Marquette University Press, 1980).

²⁸ For a defense of this point, see Thomas Flint and Alfred Freddoso, “Maximal Power,” in (eds.) Freddoso and Alexander Pruss, *The Existence and Nature of God* (Notre Dame, Indiana: University of Notre Dame Press, 1983), 81-113. Thomas Morris, “The Essentiality of God’s Goodness,” in Morris, *Anselmian Explorations* (Notre Dame, Indiana: University of Notre Dame Press, 1987), 47-52. An exceptional book exploring issues of God and modality is Richard Brian Davis, *The Metaphysics of Theism and Modality* (New York: Peter Lang, 2001).

²⁹ Swinburne 1993, 129-130.

³⁰ Augustine, *De diversis quaestionibus octoginta tribus*, 46.2. See trans. David L. Mosher (Washington D. C.: The Catholic University of America Press), 80-81. In calling this view Augustinian, I do not mean to suggest it unique to Augustine, as it is also reflected in the thought of other medieval thinkers like Aquinas (*Summa Theologiae* Ia, q. 16, a. 7, obj. 1 and reply) and Leibniz. Leibniz writes, “eternal truths, objects of the divine intellect, which constitute, so to speak, the essence of divinity itself.” See *Political Writings*, “Opinion on the Principles of Pufendorf,” trans. and ed., Patrick Riley (Cambridge University Press, 1988), 71. Robert Adams makes these connections in “Divine Necessity,” *Journal of Philosophy* 80/11 (1983): 741-752.

³¹ Thus, I side with the critical literature against Thomas Morris and Christopher Menzel’s theistic activism, a view which they misleadingly suggest as Augustinian. See their “Absolute Creation,” *American Philosophical Quarterly* 23 (1986): 353-362. For critical literature, see Brian Leftow, “God and Abstract Entities” *Faith and Philosophy* 7/2 (1990): 193-217; “Is God an Abstract Object?” *Noûs* 24 (1990): 591-598. Scott A. Davison, “Could Abstract Objects Depend Upon God?” *Religious Studies* 27 (1991): 485-497. Matthew Davidson, “A Demonstration Against Theistic Activism,” *Religious Studies* 35 (1999): 277-290. Michael Bergmann and Jeffrey Brower, “A Theistic Argument Against Platonism (and in Support of Truthmakers and Divine Simplicity),” *Oxford Studies in Metaphysics* 2 (2006): 357-386.

³² *Ibid.* Elsewhere Augustine writes, “the ideas are certain archetypal forms or stable and immutable essence of things, which have not themselves been formed but, existing eternally and without change, are contained in the divine intelligence” (*De Ideis* 2). For more on Augustine’s view, see Paul Zwier, “Augustine’s Mathematical Realism,” *Journal of the ACMS* (1989).

³³ In this sense, minimally, God’s nature is explanatorily (if not causally) prior to *abstracta*. If *abstracta* are God’s ideas, concepts, or thoughts, then there must be *some* kind of dependency relation between God’s mind and his ideas, unless one is assuming divine simplicity. However, I see no reason to think explanatory or dependency relations between metaphysically necessary beings need displace modalities. Brian Leftow remarks that metaphysically necessary beings “derive their specious plausibility from insufficiently precise understandings of alethic necessity. According to currently popular semantics, ‘*x* exists necessarily’ asserts only that *x* is to be found in every possible world. It entails nothing at all about why this is so; it leaves open the question of whether there may be some cause or causes which account

for this.” See Leftow, “A Leibnizian Cosmological Argument,” *Philosophical Studies* 57 (1989): 137. Though Leftow’s *causal* language is perhaps too strong, the point is the existence of metaphysically necessary beings is readily acknowledged as possibly *ab alio*.

³⁴ Regarding coherence, the most plausible model I have encountered is Greg Welty’s, *An Examination of Theistic Conceptual Realism as an Alternative to Theistic Activism* (PhD dis., Oxford, 2006). Regarding independent argument, see Richard Brian Davis is pioneering work in this area. See his “The Conceptualist Argument: A New Defense,” (forthcoming).