
There has been a recent wave of interest concerning the appropriate way to treat plural expressions in natural languages and Byeong-uk Yi's Understanding the Many is a welcome contribution to this wave. But before we can discuss Yi's book we shall need to examine what is at issue in the treatment of plurals.

At least when given a standard, Tarski-style, semantics, traditional first-order logic is singular. Its denoting expressions denote at most one object, its variable assignments assign a single value to each variable, and its n-place predicates are satisfied by n-tuples of objects. Natural languages, however, contain what seem to be plural terms and plural quantifiers. When we say such things as "Harold and Maude sang in unison" the subject-position in our sentence appears to be occupied by a plural term: "Harold and Maude." When we assert the Geach-Kaplan sentence: "Some critics admire only one another," we appear to be quantifying over pluralities or collections of critics rather than individual critics. Indeed a moment's consideration reveals that our language is pervaded by plural constructions that include, but are not exhausted by, conjunctive terms ("Athos, Porthos, and Aramis"), plural definite descriptions ("The three musketeers"), plural indefinite descriptions ("A trio of musketeers"), plural demonstratives ("those/those musketeers"), plural pronouns ("they"), and plural quantifiers.

How are we to handle these? Well, two basic strategies suggest themselves. We might try to handle them within the existing singular framework of first-order logic. Or we might revise our logic to allow for plural terms and quantification. The former strategy typically requires our treating what seem to be plural reference and quantification as in fact being reference to and quantification over a single entity of a special sort. Thus talk of Harold and Maude might be construed as talk of the set of Harold and Maude, or of the aggregate of Harold and Maude, or of some event in which Harold and Maude participate, or of the concept of Harold and Maude, or of the property of being-Harold-or-being-Maude, or some other single entity. Alternatively, we must revise and extend first order-logic. Theorists have typically been unhappy to do this. But in his book Byeong-uk Yi bravely argues that we must modify our logic and he sketches what such a modification might look like.

I have a great deal of sympathy for Yi's approach and I think he does a good job out sketching what a logic of plurals might look like. Yi also makes some interesting suggestions concerning how his logic of plurals might shed light upon the nature of numbers and the way a set is determined by its members. But Yi's book suffers from a number of shortcomings that are not really his fault.

Yi's book is, in fact, a collection of five essays which comprised his 1995 dissertation at UCLA. Yi tells
us that he presents these essays in their original form with merely a few stylistic changes. And, given that Yi's book is basically his dissertation, it is not surprising to find that it seems to represent his first rather than his last thoughts on the subject of plurals. As such his criticism of alternative views, development of his own view, and application of this view to issues in the philosophy of mathematics, can sometimes be seriously lacking in depth and detail. Moreover Yi neglects to discuss some of the most important literature on the subject of plurals. Admittedly some of this postdates the completion of Yi's dissertation. But not all of it does. To take two examples, it is disappointing to find no sustained discussion either of the analysis of plurals using events suggested by Higginbotham and Schein, or of George Boolos's treatment of plural quantification.

The fact that the book consists of five largely self-contained essays is also a source of difficulties. The essays tend to be extremely repetitive, repeating points made in earlier essays rather than developing Yi's ideas further. In the light of this, and especially given its high price ($60.00) and short length (103 pages), I am afraid that Yi's book is unlikely to be of terribly much use either as an introduction to the subject of plurals, or to the expert in this field. This is a shame, for I think that Yi has important and plausible things to say on this subject. I hope that in the rest of this review I can give an impression both of what is exciting about Yi's views and of what Yi still needs to do.

In the first essay of his book Yi sets out the case for treating plural terms as involving genuinely plural reference rather than merely involving singular reference to special sorts of entities. Yi's main argument, which appears again in pretty much the same form in each of the other essays, contrasts his own approach with an approach that takes the referent of "Harold and Maude" to be the set \{Harold, Maude\} and which takes plural quantifiers to range over such sets. One problem with this style of argument is that, as I noted above, the set \{Harold, Maude\} by no means exhausts the range of possible singular items that have been suggested as the referent of "Harold and Maude." Indeed one might prima facie suppose that an analysis of plurals in terms of aggregates, events, or properties, would be more natural than an analysis in terms of sets. If so, Yi's argument against the set-theoretic analysis of plurals does not address the leading rivals to his own view.

Yi's argument against the set-theoretic analysis, which he calls the Implication Argument, is basically that the set theoretic analysis of plurals commits us to the existence of sets while the original sentences do not. More precisely, Yi notes that the sentence (*) "Ezra is a critic and Thomas is a critic and Ezra and Thomas are distinct and Ezra admires only Thomas and Thomas admires only Ezra" entails the Geach-Kaplan sentence and therefore everything that follows from the Geach-Kaplan sentence but does not seem to entail the existence of sets. Hence the Geach-Kaplan sentence and the plural quantifiers it contains cannot themselves entail the existence of sets.

One problem with the Implication Argument is that, in essay 3, Yi himself analyzes the fact that John and Carol are two things as consisting in the instantiation of the property of being two things by John and Carol. But, we might protest, this analysis must be mistaken for we can run a version of the Implication Argument against it. Now “John and Carol are distinct” entails that “John and Carol are two things” and Yi seems committed to holding that this, in turn, entails the existence of properties. However, given the transitivity of entailment Yi must accept that “John and Carol are distinct” also entails the existence of properties. And, we may complain, surely the mere fact that John and Carol are distinct doesn't entail the existence of properties. In other words it seems that, if Yi's Implication Argument against the set-theoretic analysis of plurals really worked, then the same sort of argument would impugn Yi's own account.
Of course, there may be a crucial disanalogy between the two cases. But, if so, then Yi needs to make it. And, even if he can do so, this will simply bring home the point that Yi is not considering his strongest competitor. Yi does not merely need to run his Implication Argument against the set-theoretic analysis of plurals. He also needs to run it against alternative accounts which analyze plurals in terms of aggregates, events, and properties (such as the property of being-Ezra-or-being-Thomas). But at the same time Yi needs to argue that his own account is immune to a version of the Implication Argument. I doubt whether Yi can do this.

In fact the central problem with the Implication Argument, as I see it, is that our intuitions concerning the existential implications of various sentences are at best a defeasible guide to those implications. After all, one could make out a good case that the truth of "Fido is a dog" entails the existence of properties, that the truth of "I said that Fido is a dog" entails the existence of propositions, that the truth of "Fido might not have been a dog" entails the existence of possible worlds, and that the truth of "Fido peeled the onion at midnight" entails the existence of events. But who would have thought any of these things initially? In a similar way, for all that has been said, perhaps (*) does, after all, entail the existence of sets. The Implication Argument relies upon our defeasible intuitions and does not provide a sure guide to ontological reality.

Yi buttresses his claim that (*) does not entail “sets exist” by observing that the inference is invalid in elementary logic. But, while this is indeed true, it is neither here nor there. There are many fine inferences that are invalid in elementary logic. The inference from “John is to the left of Sally” to “Sally is to the left of John” is a case in point. So, for that matter, is the inference from “John and Carol are distinct” to “properties exist.” All this shows is that elementary logic needs to be supplemented in some way if it is to capture the validity of all those inferences that we intuitively count as valid.

In the second essay in his book Yi sets out a formal system of plural logic and sketches a semantics for that system. I found that Yi did a good job here and, in particular, that he did a good job explaining why he made the various choices he made setting up the logical system. My only disappointment was that Yi's semantics ends up assigning subsets of the domain to the plural constants and variables. In other words, Yi's semantics assigns plural terms and variables single set-theoretic entities as their values. This is all fine and well for a formal semantics. But it would be nice if the semantics had taken more seriously the idea that plural expressions refer to multiple objects. In order to do this, of course, we would have to replace the usual assignment function with an assignment relation which will allow us to simultaneously assign more than one object from the domain to plural constants and variables. Having done that, however, I think it should otherwise be relatively straightforward to produce a genuinely plural semantics.

In the next two essays Yi argues that numbers should be construed as properties rather than objects so that, for example, the number 2 is treated as the property of being-two. To this end Yi takes the sentence (** "Bill and Hillary are two" as being a paradigmatic arithmetical statement and then argues that this sentence makes no reference to any objects other than Bill and Hilary. In particular, Yi argues, (**) makes no reference to numerical objects of any kind. Rather, he suggests, we should understand (**) as predicating the plural property of being-two of the plural subject Bill-and-Hilary.

There is something rather appealing about Yi's suggestion here. But in the end I doubt that it works. Yi is left having to paraphrase away all apparent reference to numbers as objects and while he glosses
"2+1=3" as "If some things are two things and something is one thing that is not one of the former then the former and the latter are three things (and vice versa)" it is anything but obvious how he would gloss "0+0=0" let alone such claims as "some even number $n$ is the sum of $n-1$ primes" or "there are prime numbers whose square roots are not divisible by any perfect number." The prospects for handling such cases seem poor to me.

Yi concludes his book with an interesting discussion of how sets are related to their members. Yi considers the question: given a domain of mundane objects (ur-elements), which sets of those mundane objects are there? It is not acceptable, Yi argues, simply to answer that any collection of mundane objects is a set. This simply pushes the question back one stage to the question of what collections of mundane objects there are. Rather, Yi argues, the answer to our original question is that "any number of mundane objects forms a set" which he glosses as (***) "if there are some mundane objects then they form a set," where the quantification here is to be understood as plural. In the end, however, I don't see this as much of an advance on the answer Yi rejects. For haven't we now simply pushed the question back to that of what the possible plural values are for the plural variables bound by the plural quantifier in (***)? The obvious answer, every possible collection of mundane objects is a possible value for a plural variable, just brings us around in a circle. So I don't think Yi's suggestion gets us any further, in the end, towards answering the question Yi sets out to answer.

To summarize, Understanding the Many is a brave attempt to argue for and implement logical revision which takes up some interesting issues in the philosophy of mathematics along the way. Given its nature as a dissertation, however, it has neither the breadth nor the depth to deal with the issues surrounding plurals adequately. So I hope that Yi is planning a more detailed and comprehensive monograph developing his ideas. For I certainly think they are worth development.

Anthony Everett
Illinois State University

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