Review of “Frege's Logic”

A. M. Ungar
SUNY, Albany

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More than twenty years ago Michael Dummett lamented the fate of Frege’s work, that after long neglect it should have become a battle ground for exegetes: “each [new book or article] declaring that every previous exponent has misunderstood Frege from start to finish” (The Philosophical Quarterly, 1984, p.194). Reading Frege’s Logic brings his words to mind for it argues that previous commentators, viewing Frege’s logical work through the distorting lense of modern logic, have failed to see that his Begriffsschrift is not an early, albeit eccentric, formulation of quantification theory, but something radically different and more interesting.

As is well known, in Begriffsschrift Frege introduced a formal system of logic. His primitive symbols include a judgment stroke, a conditional stroke, a concavity with associated gothic letter, and Latin italic letters. The judgment stroke, attaching to the left of every formula in a Begriffsschrift deduction, is supposed to indicate that the formula in question is being asserted as true; it has been abandoned by later logicians. The other symbols in the list have usually been identified with modern analogues: the conditional stroke is treated as a symbol for material implication, the concavity as a universal quantifier, Gothic letters as bound variables and Latin italic ones as free variables. These identifications are all questioned by Macbeth. A noteworthy feature of Frege’s notation is that the conditional stroke is a two-dimensional symbol: placed to their left, it joins two formulas arranged in a column, the consequent above the antecedent—or the main component above the subcomponent as Macbeth, following Frege, prefers. Frege set great store by his distinctive notation, as does Macbeth. It yields formulas composed of a column of logically simple clauses flanked on the left by an array of horizontal and vertical strokes (with, perhaps, one or more concavities, each cradling a Gothic letter) which could be said to picture the logical relationships holding among them. Because it is too difficult to reproduce Frege’s notation in the present context, however, I have resorted to the clumsy expedient of describing rather than displaying formulas and, on one occasion, have used more conventional linear notation with an appropriate warning.

In her introduction, Macbeth notes certain characteristics of Frege’s logic which seem anomalous from the perspective of modern quantificational logic: his insistence on using a two-dimensional notation for the formulas of his Begriffsschrift, for example, his use of the judgment stroke with the requirement that a genuine inference can only be drawn from true premises, and his use of different kinds of letters for free and bound variables. In the remaining five chapters of her book, she develops an alternative interpretation of Frege’s logical language—one that, she promises, will transform these apparent anomalies into logically justified features—tracing the development of his ideas from the early Begriffsschrift (1879) to his ill-fated introduction of basic law V in the Grundgesetze (1893, 1903) and
Two key claims underlie Macbeth’s new interpretation. The first is that: “Frege’s logic is founded on a distinctive conception of law as a principle that governs inference; [these] have the form of rules according to which to reason as they contrast with premises from which to reason.” (p.177). The second is that: “the content of a [Begriffsschrift] sentence must, for the purposes of judgment and inference, be variously analyzable, that only relative to an analysis can [its] truth conditions be formulated.” (p.2)

The first claim is taken up in chapter 1. There she argues that Begriffsschrift generalized conditionals, i.e. formulas constructed using the conditional stroke, and with Latin italic letters to convey generality (Macbeth resolutely refuses to call them variables), have a special importance. They are to be interpreted as expressing laws which govern inferences. A simple example of the form such a law might take (in linear notation, where “ ” is supposed to do duty for Frege’s conditional stroke) is |– Fa Ga. In contrast to a universal sentence of quantificational logic, this is not to be interpreted as making any sort of factual claim; it is a rule by which to reason, an inference license which allows the conclusion Go to be drawn from the premiss Fo, for any particular object o. Her contention is that generality, as it figures in the statement of such laws, is fundamental to Frege’s overall conception of logical generality.

Macbeth then turns to the second claim. She considers first atomic sentences and truth functional combinations thereof and then formulas involving Latin italic letters (chapter 2), arguing in each case that a Begriffsschrift sentence can be analyzed into function and argument in various different ways, that such analyses reveal the inferential role the sentence can play and that, prior to such an analysis, the sentence merely displays objects and concepts in various logical relations. To take a simple example from the language of arithmetic: “Only relative to an analysis that identifies some number(s) as argument(s) and the remainder as the function is the sentence [‘2^4 = 16’] so read correctly described as saying something about something. Independent of an analysis,... it merely exhibits three numbers in an arithmetical relation.” (p.43) This idea is extended to truth functional compounds by reading the combination of strokes (to express the conditional and negation) which Frege writes to the left of the individual clauses in such a formula as a compound sign for a logical relation and interpreting the compound itself as merely displaying truth values in a logical relationship and thus analyzable in various ways. (Worth noting is that, on Macbeth’s reading, unanalyzed nested conditionals may lack a unique main connective: for example, the same Begriffsschrift formula can be analyzed as a conditional with antecedent P and consequent another conditional, with antecedent Q and consequent R, and as a conditional with antecedents P & Q and consequent R.) Macbeth goes on to argue that, as employed by Frege, Latin italic letters in generalized conditionals are not to be interpreted as free variables. They attach to the conditional stroke itself, rather than to the concept words occurring as antecedent and consequent. On Macbeth’s account, replacing an object name by a Latin italic letter is more akin to abstraction than to quantification: it “pushes things up a level” so that what was a sentence about objects (“If o is F, then o is G”) becomes one about concepts (“The concept F is subordinate to the concept G”).

This leads to the question of how to interpret Frege’s concavity notation (chapter 3). For Macbeth, it is Latin italic letters that serve as Frege’s primary means of expressing generality and she rejects the idea that his concavity notation should be read as a universal quantifier. While she seems not to deny that it was introduced by Frege to delimit the scope of the generality in a formula, its significance for her lies in its expressive function: to form names for higher level concepts and relations. Her understanding of the difference between a generalized conditional written using Latin italic letters and one using the concavity

notation with Gothic letters, for example, is that the former displays a relationship between two concepts, namely that of subordination, whereas the latter contains a name for the (second level) relation of subordination, formed from the conditional stroke together with a concavity, and says that the concepts in question fall under this relation. (By the same token, it displays a third level relationship between subordination and two first level concepts.) Macbeth describes a process according to which, subsequent to Begriffsschrift, Frege came explicitly to recognize that concepts were something in their own right, which could themselves appear as arguments of other concepts—hence their stratification into levels: first level concepts taking objects as arguments, second level concepts taking as arguments first level concepts, and so on. She goes on to argue that he finally arrived at a view of logic according to which its subject matter was the higher level concepts and relations expressible in the language of Begriffsschrift, and its task was to formulate laws governing the lower level concepts and relations which fall under them.

Macbeth sees the distinction between Sinn (sense) and Bedeutung (meaning or reference) as central to Frege’s logical theory. In her fourth chapter, she gives an account of how difficulties with the notion of content employed by Frege in his early writings led him to draw this distinction. Once it is in place, she is in a position to claim that it is the sense or cognitive content of a sentence, not its truth conditions, that is pictured by the symbolism of Begriffsschrift, “its content as it matters to the correctness of judgments and inferences it can figure in” (p.136). This sense is fixed by the rules of the language, both formation rules and rules of inference, and by rules governing the application of its signs. To grasp the sense of a sentence, it is not enough to know what would be the case if it were true, “one must also know what follows from it with or without auxiliary premises” and it is this knowledge which Frege makes explicit for a fragment of language in his Begriffsschrift. She concludes that sense attaches to an expression only relative to a whole language so that, “on Frege’s mature account, it is the whole language that is the minimum unit of cognitive significance.” (p.138)

In her final chapter, Macbeth discusses the difficulties associated with Basic Law V, the comprehension axiom for courses of values of functions (or extensions of concepts) which Frege laid down in the Grundgesetze and which generates an inconsistency. The gist of Macbeth’s argument here is that, while Frege’s logicist program required the introduction of extensions, objects, concepts and relations suffice for the fundamental part of his logic which can thus avoid inconsistency.

Macbeth declares that she aims to defend her reading textually and she is careful to cite Frege’s writings in support of each of her claims. Certainly, her arguments are not to be lightly dismissed, but it is not possible to do them critical justice in a brief review. In what follows I try merely to indicate why I do not find her interpretation altogether convincing.

1) Frege’s words are not always able to bear comfortably the weight she places on them. For example, Macbeth attributes to Frege the view that because “All our knowledge is essentially mediated by an inherently historical learned public language” (p.153), it follows that even the laws of logic may be called into question so that we cannot have certainty about them. Her evidence is a passage from the introduction to Grundgesetze in which Frege remarks that, while it may be impossible for us to reject the law of identity, that does not hinder us from supposing that there are beings who do. For Macbeth, this means that Frege can imagine that there could be a reason for calling this law into question. The remark occurs in a passage in which Frege characteristically attempts to distinguish his position from that of psychological logicians. His point is that whether we can or cannot reject the law of identity is a psychological not a logical question (one about which he remains neutral). It allows that we can imagine
explanations for why someone might hold or reject the law, but says nothing about the possibility of reasons for doubting it.

2) Macbeth’s use of Frege’s texts is highly selective, something about which she is refreshingly candid. For example, one of her more controversial claims is that Latin italic letters do not function as free variables (with the generality interpretation) and the concavity with Gothic letter is not to be interpreted as a universal quantifier. She concedes that: “Frege does often describe this notation in ways that suggest [that they do],” but adds: “he also often describes that notation in ways that are incompatible with that reading. It is the latter passages we focus on here.” The reason given is that: “all texts considered, it is the latter passages... that seem best to reflect Frege’s most considered views.” (p.63) While I haven’t attempted a count, my impression is that there are many more of the former than of the latter however. Furthermore, passages supporting a quantificational interpretation are usually quite unambiguous, whereas the passages which Macbeth cites can often, with a little charity, be read as compatible with it.

3) I suspect that this sort of piecemeal sniping would strike Macbeth as somewhat beside the point. What she sets out to offer in this book is a comprehensive reinterpretation of Frege’s logic which, taken as a whole, is superior to more standard interpretations. Why then should one be dissatisfied with what she calls the standard interpretation? Macbeth seems to suggest that the various anomalies listed in her introduction suffice to discredit it—Macbeth calls them mistakes from the perspective of quantificational logic—but this is surely an exaggeration. Some are scarcely problematic at all. For example, Frege’s reason for using both Latin italic and Gothic letters—that they function differently in inference—as explained in Grundgesetze, §15, is not incompatible with their interpretation as free and bound variables, respectively. Likewise, his remarks about the superiority of his two dimensional notation, pace Macbeth’s reading, do not present any real problem for a quantificational reading of his logic. Others, like the judgment stroke, have disappeared from modern logic and, while this certainly indicates a divergence between Frege’s ideas about logic and ours (the inevitable result, one would suppose, of a century of further logical investigations) it poses no particular problem for the quantificational interpretation of his logic as such. Furthermore, even though Macbeth is able to incorporate these features into her own reading, they offer less support for it than that may suggest. The logical justifications for using two kinds of letter and a two dimensional notation cannot plausibly explain why Frege adopted them since, according to Macbeth, these justifications depend upon later developments in his thought. As for the judgment stroke, although she provides (on p.26) a good reason why it is needed in Frege’s logic as she understands it, to the best of my knowledge the reason is hers and is nowhere to be found in Frege.

4) Macbeth rightly suggests that posterity has introduced its own biases into its reading of Frege—inevitably, those aspects of his work which resonate with us are emphasized at the expense of those which no longer seem fruitful—and she performs a service by reminding us that there are overlooked aspects of Frege’s work that may deserve our attention. But this is not to concede that the picture she draws is any less distorted than the one she hopes to supplant. Although the fact is downplayed in her book, her interpretation of Frege is clearly influenced by Robert Brandom’s program of inferentialism in semantics and expressivism in logic with the result that she gives special emphasis to those ideas of Frege’s that can be related to this program (the importance of inferential role in the determination of Sinn, for example, and the aim of Begriffsschrift to make explicit the laws governing our inferential practice). The overall coherence of her interpretation seems to me to derive more from this source than from anything to be found in Frege’s texts themselves.
To conclude, Macbeth has written a stimulating and original book that will interest all students of Frege’s work. It will lead them back to Frege’s own writings, either to confirm views they already hold or to modify them in the light of Macbeth’s reading. In either case, their understanding will be deepened.

A.M. Ungar
SUNY, Albany