Res Cogitans

Volume 2 | Issue 1 Article 23

7-30-2011

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Res Cogitans (2011) 2:215-228 2155-4838 | commons.pacificu.edu/rescogitans

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Published online: 30 July 2011

Abstract

In 1977 when Appel, Haken and Koch used a computer to mathematically solve the century old four-color-problem philosopher Thomas Tymoczko thought that the epistemic justification in mathematics had been changed. Essentially, Tymoczko, and others, argue we can now have mathematical epistemic justification through *a posteriori* means. This has obvious implication in philosophy of mathematics and epistemology because this would be the first case where mathematics isn't justified through *a priori* means of investigation. However, I ultimately disagree with Tymoczko. I argue that computer-aided-proofs still warrant an *a priori* means of justification. In order to show this, I refer to advances in philosophy of mind, mainly, the extended mind thesis.). I will argue that our mind has evolved to enter into symbiotic relationships with non-organic entities in order to offload certain internal capacities. I believe that this is what constitutes humans amazing gift of rationality and intelligence. Thus, when we use a computer-aided-proof to solve unsurveyable proofs, we are really extending our minds into these cognitive tools and extending our method of proof checking to be more efficient and quicker. Thus, the *a priori* is saved because the computer is just a part of the causal cognitive loop that constitutes our mind.

I. Introduction

In 1977 Appel, Haken and Koch devised a computer proof that solved the four color problem, a problem that was once thought to be too long to solve (Appel, Haken and Koch, 1977). They showed that every map on the plane or sphere could be colored with no more than four colors in such a way that neighboring regions are never colored alike. The significance of this theorem is that they used a computer-aided-proof to check their results, due to the fact that it would take lifetimes to survey the whole proof.

While the four-color-theorem (4CT from here on out) was solved mathematically, it raised many philosophical questions. Most notably, this was potentially the first time that empirical methods have been inserted into mathematics and the results of the theorem may be justified through a posteriori means rather than a priori means. The most notable voice on this side of the debate, Thomas Tymoczko states, "“Computer proofs bridge the great gap philosophers have seen between mathematics and natural science. If traditional philosophy is right, then mathematicians should not accept computer proofs" (Tymoczko, 1981, p.123).

However, I ultimately disagree with the conclusion that Tymoczko draws and feel that we are still warranted, through a priori means, in accepting computer-aided-proofs. In my defense of computer-aided-proofs I will try something unique and refer to the extended mind thesis, as made famous by Clark and Chalmers (among others). I will argue that our mind has evolved to enter into symbiotic relationships with non-organic entities in order to offload certain internal capacities. I believe that this is what constitutes humans amazing gift of rationality and intelligence. Thus, when we use a computer-aided-proof to solve unsurveyable proofs, we are really extending our minds into these cognitive tools and extending our method of proof checking to be more efficient and quicker. Thus, the a priori is saved because the computer is just a part of the causal cognitive loop that constitutes our mind.

I will divide this essay into three sections. In the first I will explain the philosophical significance of the 4CT and the consequence that Tymoczko (and others) raise. In the second section I will give an in depth account of the extended mind and how it arrives through extended cognition. Finally, I will show how the extended mind refutes Tymoczko's worries and preserves the a priori justification of the proof.

II. The Philosophical Significance of the 4CT

In his 1979 essay, Tymoczko explicitly states his philosophical concerns about using computer-aided proofs (for the rest of this essay I will refer to them as CAPs) and he distinguishes two unique consequences to the philosophy of mathematics and epistemology. First, he worries that, "if we accept the 4CT as a theorem, we are committed to changing the sense of a 'theorem', or, more to the point, to changing the sense of the underlying concept of a proof" (Tymoczko, 1979, p. 58). Second, "use of computers in mathematics, as in the 4CT, introduces empirical experiments into mathematics," which "raise again for philosophy the problem of distinguishing mathematics from the natural sciences" (Tymoczko, 1979, p.58). It's easy to see the philosophical problem that arises with the 4CT because in the history of philosophy of mathematics, it's widely believed that all mathematical truths can be known by a priori methods of investigation, and that if one believes something a mathematical proof to be true, they will then have a priori knowledge of the proof in question.

With the two consequences from above, arise four separate problems, which some philosophers, Tymoczko included, believe would change the underlying concept of a mathematical proof. This would then allow for a posteriori methods of mathematics. The most profound problem that arises is that CAPs are that they are unsurveyable by humans. Tymoczko argues that, “Proofs are the guarantees of mathematical knowledge and so they must be comprehended by mathematicians. A proof is a construction that con be looked over, reviewed, verified by a rational moral agent” (Tymoczko, 1979, p.58). Another defining characteristic of a proof, as defined by Tymoczko, is that proofs are formalizable. Untrustingly, these parameters of what makes mathematical proof work together, and Tymoczko himself says they are “two sides of the same coin”.

Tymoczko is not the only philosopher to believe there is a problem with accepting that CAPs lead to a priori knowledge. Philip Kitcher voices a similar concern about unsurveyable CAPs when he says, “[CAPs] are so long that they cannot lead us to a priori knowledge” and again by concluding that, since CAPs cannot warrant a priori knowledge because humans don’t have the cognitive capabilities to recall each individual step (Kitcher, 1983, p.46).

Tymoczko, among others, believe that the problem with a proof being unsurveyable is that it cannot be checked for errors that may or may not be contained inside of the proof. I believe philosopher Paul Teller recapitulated Tymoczko’s conclusion well by saying, “the computer proof of the combinatorial lemma is subject to error: computers can make mistakes” (Teller, 1980, p.798). For Tymoczko, the potential for computer errors distinguish CAPs from established a priori proofs. It’s clear that Tymoczko believes, that any CAP that is unsurveyable could potentially suffer from either an internal malfunction, whether it is a hardware or software error. Mark McEvoy condenses this argument against CAP warranting a priori knowledge best when he said, “Since the proof cannot be surveyed, w cannot rule out the possibility that some of the inference that the computer has carried out over the course of the proof are not a priori sanctioned moves, but are instead errors caused by a computer malfunction. Since we cannot rule out this possibility, our CAP cannot be a traditional a priori proof” (McEvoy, 2008, p. 381).

Moving on to the latter consequence of CAPs that Tymoczko puts forth (CAPs introducing empirical methods into mathematics), it becomes apparent in the relevant literature that, even if one looks past the surveyability problem, there is still the worry that by petitioning to computers to solve mathematical problems we insert an inherently empirical aspect to the proof at hand. If there were an empirical nature to CAPs then there would certainly not fit the definition of a priori. For example, A.J. Ayer states that, “Empiricts must deal with the truths of logic and mathematics by… say[ing] that they have no factual content, and then he must explain how a proposition which empty of all factual content can be true and useful and surprising” (Ayer, 1956, p. 72). However, certain philosophers see CAPs depending on empirical knowledge about computers. This is an essential part to Michael Resnik’s (1997) argument, which is that the epistemic warrant provided by CAPs are contingent upon the computer functioning appropriately. To Resnik, an appeal of this nature is inherently empirical.

Similarly, Kripke introduces an example that illustrates this worry. He argues:

> No one has calculated or proved that the number is prime; but the machine has given the answer: this number is prime. We, then, if we believe, that the number is prime, believe it on the basis of our knowledge of the laws of physics, the construction of the machine, and so on. We therefore do not believe this on the basis of purely a priori evidence. We believe it on the basis of a posteriori evidence (Kripke, 1972, p. 35)

Again, Kripke is showing that, without consulting on how the computer came up with the answer and with no knowledge of how the computer obtained the answer, our epistemic warrant (in believing that the number is prime) relies in large part on the construction of the machine itself. I would have to agree with Kripke here, but as I will show later, this is not the case with CAPs because the proof-checking method used by the computer is the same method the mathematician/logician would employ.

If we hear the worries voiced by the likes of Tymoczko, Kripke and Resnik, we can conclude that, if a computer introduces empirical content into a CAP, then the CAP is an experiment. Thus, this contradicts and prior definition of the a priori. Tymoczko writes:

> In fact, there is no surveyable proof, no proof in the traditional sense, of the 4CT, nor is there likely to be one. Still Appel, Haken, and Koch's work provides mathematically convincing grounds for the 4CT. What can be surveyed, what is presented is their published work, is like a mathematical proof where a key lemma is justified by an appeal to the results of certain computer runs or, as we might say "by computer." This appeal to computer, whether we count it as strictly a part of a proof or as a part of some non-proof-theoretic component of mathematical knowledge, is ultimately a report on a successful experiment. (Tymoczko, 1979, p. 63)

Before I go on to explain the tenants of extended cognition and the extended mind, I want to recap the philosophical problems that Tymoczko and others see with the 4CT and other CAPs. To recap, the first problem that Tymoczko raises is that, since the 4CT is too long to be completed in a human lifetime and a computer has to solve it then the proof is unsurveyable. Of course, for Tymoczko, a proof must be surveyable, this way a mathematician or logician could check for any embedded mistakes that were made due to a hardware/software malfunction by the computer. Thus, CAPs are not justified by a priori means.

One claim that seems to be embedded inside all of the aforementioned arguments on why CAPs warrant a posteriori knowledge is the claim that pure mathematics and logic, which are warranted by a priori means, is an inherently human task. For example, a big cog inside of Tymoczko's argument is that a proof must be surveyed, formalized and checked for error by a rational agent. As Teller puts it, "since he holds that proofs must be surveyable and then assumes without any argument that 'surveyable' must mean unaided human surveyability" (Teller, 1980, p. 802). As I will show later, it is a basic proviso of human rationality and intelligence that we (humans) have evolved to extend our cognition and mind into non-organic objects to modify our capacities. Thus, it's not the computer that is solving the proof by another means of proof solving, but rather a human extending their mind into the computer to solve, by the same methods, a proof that would be to long to ever carry out by hand.

III. The Origins and Significance of Extended Cognition and the Extended Mind

Before I go on to explain how the extended mind solves refutes the philosophical worries raised by Tymoczko about the 4CT, I will first provide a sketch of what exactly extended cognition is and how it leads to an extended mind.

As a preface to the extended mind, I should note the origins of the extended cognition. While Clark and Chalmers coined the phrase (extended mind) and wrote explicitly on the subject in 1998, many others had been talking about it for quite some time, including: Clark, Dennett, Chalmers and Hutchins. However, in throughout the discussions of extended cognition, there is an idea that lends itself quite well to the discourse that often gets overlooked. Neurophysiologist William Calvin came up with the novel and highly useful idea of a Darwin Machine (Calvin 1990). Owen Flanagan summarizes a Darwin machine nicely by saying, "The brain is a biological device in which pressures to achieve adaptive coordination between the organism and the external world operate to select populations of neurons, from predisposed groups, to recognize certain patterns in the external world and within the body itself and to activate certain appropriate motor routines in response to patterns recognized" (Flanagan, 1992, p.40).

A key premise of a Darwin Box that I think is relevant to human cognition is that higher-level thought and problem solving involves processes comparable to that of natural selection. Flanagan claims, "We select approaches to problems that have worked in similar situations in the past. In utterly novel situations, we spin out novel problem-solving strategies by mixing previous solution strategies, creating "idea mutations" as it were. We try some of these, and if they get the job done, there is selection in their favor in the future" (Flanagan, 1992, 40). The point that needs to be absorbed here is that when we confront tasks or a problem that needs to be solved, our brain adapts so that we can decode the problem and later solve it in the most efficient manner. So it seems fair to say that when we are confronted with mathematical problems or proofs that are too tedious and we are not able to solve them with our inherent on-board cognitive capacities, we then adapt and use tools to solve them.

While a Darwin Box most certainly plays a role in human cognition for the connectionist and most externalists, it still doesn't quite paint the picture that human rationality is contingent upon extended cognition. Clark and Chalmers believe that in a coupled system some functions that play an active causal role in the cognition are located outside of the skull. What is essential here is that they are playing the same causal role that, were it located inside the skull, would be considered an active player in the cognitive process. If the external component were removed, the system's behavioral competence would drop, just as if you removed part of the brain. These types of actions are referred to as epistemic actions. Clark and Chalmers say, "Epistemic action, we suggest, demands spread of epistemic credit. If, as we confront some task, a part of the world functions as a process which, were it done in the head, we would have no hesitation in recognizing as part of the cognitive process, then that part of the world is part of the cognitive process" (Clark and Chalmers, 1998, p.644). Thus, their thesis is that this kind of coupled process counts equally well as a cognitive process, whether or not it's wholly in the head.

Hopefully the link between extended cognition and the Darwin Box are clear. Overtime and in order to solve more complex problems humans have needed to extend their cognition beyond the skull. Clark and Chalmers go on to suggest a similar sentiment when they say, "It certainly seems that evolution has favored on-board capacities which are especially geared to parasitizing the local environment so as to reduce memory load, and even to transform the nature of the computational problems themselves. If so [evolution finding it advantageous] then external coupling is part of the truly basic package of cognitive resources that we bring to bear on the world" (Clark and Chalmers, 1998, p.646). Here extended cognition, and also the extended mind (as we will see later), share similarities to Richard Dawkins idea of an extended phenotype. Dawkins' extended phenotype is the idea that the boundary is blurred between the organism and the environment in which they must act (Dawkins, 1982).

An important question and aspect of extended cognition that needs to be answered and explained in greater detail is how are humans able to form symbiotic relationships with objects in the environment in order to offload capacities and achieve greater accomplishments with our cognitive abilities? For externalists such as Dennett and Clark the answer is simple, language. Dennett argues that, "Minds are composed of tools for thinking that we not only obtain from the wider world, but largely leave in the world, instead of cluttering up our brains with them" and "the prosthetic extension of our brains that permits us to play such glorious tricks with time: we call language" (Dennett, 2000 p.24). What Dennett is implying here is a similar sentiment to that of Clark and Chalmers (1998), but he is adding the element that language is the vessel that allows for extended cognition and thus, human intelligence to reach the levels that it does. This is also an interesting quote by Dennett because of his attention to resource allocation (time). Clark also shares a similar sentiment with the way language aids extended cognition in terms of lifting time constraints that are placed on us (Clark, 2000). We can draw the connection here to extended cognition and the use of computers. When humans enter into a symbiotic relationship with a computer, the functions of the computer allow the person to extend their cognition far beyond what they could before. Thus, a person is allowed to solve a proof that would take lifetimes in a mater of moments. However, it needs to be made explicitly clear that it's still the human, powered by their cognition, solving the problem. The computer is not adding something new that the human is unaware of, like in the Kripke example from section II. Since the computer is in a symbiotic relationship with a human, the computer is programmed to solve the problems the same way the person would.

It's easy to think of ways in which we offload certain cognitive capacities by forming a coupled system with non-biological organisms. Some examples are using a pen and paper to take notes, a calculator for a complex calculation, or an artist using a sketchpad to sketch his or her draft. It seems that almost every computational problem we do involves us entering a symbiotic relationship and it should the question should then be asked, is it possible for humans to not extend their cognition? For Dennett, Clark and myself the answer is simply, no. Dennett makes this claim multiple times. He says, "The Primary source [of our greater intelligence] is our habit of off-loading as much as possible of our cognitive tasks into the environment itself- extending our minds (that is, our mental projects and activities) into the surrounding world (Dennett, 1996, p. 135) and "It is this distribution of the tasks of intelligence that makes our minds so much more powerful than all other animal minds." (Dennett, 1993, p.27). Clark also adds, "Such capacities [capacities for advanced reason] depend heavily upon the effects of a special kind of hybridization in which human brains enter into an increasingly potent cascade of genuinely symbiotic relationships with knowledge rich artifacts and technologies" (Clark, 2000, p.122).

However, Clark doesn't stop just with language and extended cognition into the environment. Clark also insists that human cognition is contingent upon using technology as cognitive tools (also referred to as cognitive aids). He says, The central idea is that understanding what is distinctive about human thought and reason may turn out to depend on a much broader focus than that to which cognitive science has become most accustomed: a focus that includes not just body, brain and the natural world, but the technological props, aids and scaffoldings (pens, papers, PC's, institutions, etc.) which our biological brains learn, mature and operate (Clark, 2000, p.131).

Now we that I have traced the lineage of extended cognition and have shown the reasons for believing why in order to have advanced cognition, we must extend our cognition into the environment and form symbiotic relationships with cognitive aids (technology), I can now show how extended cognition leads to an extended mind. . Clark and Chalmers use this idea that cognitive processes can be coupled systems with the environment to show that there are cases where external factors make a significant contribution to some mental states. They are then able to advance their thesis further to say those beliefs can be constituted partly by features of the environment when those features play the right role in cognitive processes. Clark and Chalmers see the extension of beliefs into the environment as a step further (to extending the mind) than the mere extension of cognitive processes. Their final step is to extend the self beyond the skull (on the basis of extending beliefs etc. beyond the skull).

In order to show that these coupled cognitive processes play the right role to extend mental states such as belief into the environment, Clark and Chalmers developed the famous thought experiment of Otto, the Alzheimer's patient. Otto carries around a notebook and when he learns new information, he writes it in the notebook. Conversely, when he needs old information, he retrieves it from the notebook. In Otto's case, his notebook plays the role that is usually played by biological memory. Today, we know that memories are stored first in the medial temporal system and are then distributed across networks in cortical regions.

If Otto wants to go to the MOMA he consults his notebook, which tells him the museum is on 53rd St. Clark and Chalmers insist that Otto walked to 53rd St. because he wanted to go to the MOMA and he believed the museum was on 53rd St. With this, it's reasonable to say that Otto believed the museum was on 53rd St. without ever consulting his notebook. The notebook plays the same role that memory plays for a normal functioning human. While there is a little difference between the cognitive role of memory and the holding of a belief, the difference is an important one. What Otto sees in the notebook he believes, but how does Otto believe what's in the notebook? The belief rests on memory: if the memory is in the notebook, then Otto believes it. The cognitive role of memory gives rise to beliefs. The information in the notebook acts just like the information that constitutes non-occurrent beliefs (beliefs that are not currently being entertained by the subject). Non-occurrent beliefs and memories constitute most of our beliefs and memories. Inside of the brain, non-occurrent memories are recalled under certain circumstances, an example being when someone is asked a question. The person being questioned then recalls that memory (answer) and a particular region in the cortex plays an appropriate role in the recall. Levy says that spelling out what kind of role is appropriate is a difficult task, but he does say, "We can say that something counts as part of a subject's memory if the role it plays in recall is causal, and if it encodes information that is activated in the recall. The cortical regions identified by neuroscientists clearly satisfy these conditions". However, regions of the world that are external to the skull, such as Otto's notebook, satisfy these same conditions. Thus, non-occurrent mental states can be stored externally.

Clark and Chalmers add a set of criteria to be met by non-biological objects for inclusion into an individual's cognitive system. These are:

(1) It has to be a constant in the person's life
(2) The information in the extended object is directly available to the person without difficulty
(3) When the information is retrieved from the extended object, the person automatically endorses it
(4) The information in the extended object has been consciously endorsed at some point in the past (however it should be noted that the authors said this point is debatable)

Here we can see that Otto's notebook fulfills all of these criteria. With this, Clark and Chalmers can argue that since Otto's notebook plays the same functional role in memory retrieval as neural states play in a person's memory, and that the notebook fulfills the criteria listed above, we should say that Otto's notebook ought to count as part of a coupled cognitive system. Thus we can infer that Otto's notebook is a part of the parity principle, which is the following: if something plays a role in cognitive activity, for example if it were it internal, then we would have no difficulty in concluding that it was part of the mind, it should be counted as part of the mind whether or not it is internal.

IV. The Extended Mind and its Answer to Tymoczko's Worries

Hopefully in my lengthy explanation of the importance of the extended mind (and cognition) a connection has been made between the symbiotic relationship a mathematician's (or logician's) mind forms with a computer when they are solving a theorem using a CAP. In this section I will take each of the consequences and worries Tymoczko has and address them individually.

The first worry that I want to expunge (the unsurveyable nature of CAPs) will help resolve the worry that computers are fallible. In his response to Tymoczko, Teller almost accidently answers the surveyability problem with the extended mind. Teller says, "It is a characteristic [surveyability] which some proofs have. In particular, we may take advantage of new methods of surveying [such as using cognitive tools] as long as these enable us to meet sensible demands on checking proofs, and a shift in the means of surveying actually used means only a shift in methods of checking proofs, not a shift in our conception of the things checked. Different mathematicians survey and check proofs differently, and they have long used aids such as pencil and papers, and sometimes tables, slide rule, or calculator" (Teller, 1980, p.798). I think Teller is on to the idea that mathematicians have been forming cognitive symbiotic relationships with non-biological entities to solve theorems throughout history.

Over time our relationships with cognitive tools has become more and more advanced, resulting now with computers that can check extraordinarily and impossibly long proofs. When we extend our mind into a computer to solve a proof, we are still using the same methods to solve the proof if we were writing it out, hence we are also using the same method of proof-checking. I would agree that if the computer were checking the proof differently, then there would be a potential spot for contention. However, the computer is using the same methods that humans use, just at an accelerated rate. Thus, if we are using entering into a cognitively coupled relationship with the computer, meaning the proof can only be solved with both the human mathematician and the computer, and we are having the computer check the proof by a priori justified methods. Thus, we are warranted in having a priori knowledge of the result. The computer serves nothing more than an admiration in how far our cognitive capacities have evolved and what it has allowed for us to be able to solve. Ed Hutchins exemplifies this idea by saying, "[Such tools] permit the [users] to do the tasks that need to be done while doing the kinds of things people are good at: recognizing patterns, modeling simple dynamics of the world, and manipulating objects in the environment" (Hutchins, 1995 p. 155).

I can draw this point out even more. Suppose that there is a world, we'll call it World X, that is identical to ours in everyway except that everyone has knowledge on the rudiments of first order logic. In this world, they too are concerned with the surveyability of a proof, so, when confronted with a proof similar to the 4CT (in terms of length) they divide up the proof so that each person surveys and checks a segment. With everyone in the world surveying, the task now went from taking a few lifetimes to survey by hand to taking as long as a computer to survey and check the entire proof.

Conversely, in our world, a single logician is working on the same proof as World X. However, he realizes that the proof is too long to be checked by hand and sends the proof through a computer program to be checked. It should be noted that the computer is using the same proof-checking method as the organic proof-checkers in world X. At the same time, both proofs are solved with the same valid answer. Does this mean that the people in world X have a priori knowledge of the proof, while the lone logician in our world doesn't? Certainly not, both are worlds have a priori knowledge of the proof, they just used different means of checking it. I go as far to conclude that the lone logician in our world has a more efficient system because he has less chance of error by using a computer to expedite his proof checking methods. I'm warranted in believing this because there is more room for error by the people on planet X, since there are billions of people checking the proof, there are billions of variables that could go wrong. Also, it is evident that the lone logician in our world has a higher evolved mind (remember the Darwin machine) because he formed a more efficient symbiotic relationship with the computer. Don't forget that all of the people on Planet X are extending their minds amongst each other to solve the proof too, just not as efficiently has the lone logician.

In a piece by Dennett, he draws a similar parallel to externalizing labels. Dennett writes, "A check mark is a way of making the world simpler; it cuts down on your cognitive load giving you a simple perceptual task in place of a more difficult perhaps impossible task" (Dennett, 1993, p.545). The similarity here is that one can make external labels to offload memory capacity and turn an impossible task of memorization into a relatively easy one. However, it should be made clear that the internal form of label making is the same as the external form (with the obvious exception that one label is internal and one is external). I believe that this is analogous to the lone logician in the example above. The lone logician took an internal method of proof checking, which would be impossible to carry out, and made it possible by externalizing into a computer, the exact same method of proof-checking.

In my thought experiment, I also hope to have answered the worry that computers are fallible and could make a mistake without the mathematical or philosophical community catching. If the logician/mathematician's proof-checking procedure is correct, then what comes out of the computer should also be correct and vice versa if it's wrong. I understand the worry that you might not know if the computer malfunctioned, but I believe like any good mathematician or logician, they would check their answer on another computer. If there is a flaw in the software, then, again, there is a flaw with proof-checking method of the mathematician of logician who wrote it. Famous externalist Tyler Burge makes a similar point when he says, "The meaning of the machine's activities, and indeed the nature of its rational powers, are derivative from intentionality and rationally of rational beings. Such activities provide an amplification of the designers' and programmers rationality" (Burge, 1998, p.9). Thus, if there is a failure in the software, it's a failure in the rationality of the programmer and their methodology.

Tymoczko raises the objection that, "Some people might be tempted to accept appeal to computers on the ground that it involves a harmless extension of human powers. On their view, the computer merely traces out the steps of a complicated formal proof that is really out there. In fact, our only evidence for the existence of that formal proof presupposes the reliability of computers" (Tymoczko, 1978, 72). Tymoczko then goes on to say that our knowledge that computers are reliable can only be justified empirically. However, I don't think this objection is that relevant. The point can, and was made by Burge (1998), that the only way we know our brains are reliable is through an empirical methodology. Certainly Tymoczko wouldn't say that since we know, through a posteriori means, that our brains are reliable that makes our knowledge of simple proofs a posteriori too.

In regards to the worry that CAPs introduce empirical methods into mathematics, thus turning them into experiments I think Tymoczko misunderstanding what constitutes a scientific experiment. As I have shown before, just because we know a computer is reliable through a posteriori methods doesn't mean that CAPs warrant a posteriori results if we are entering into a coupled system with them and using the same methodology we would use internally. As I quoted from Ayer earlier, a priori justifications must produce no factual content, and surely there are CAPs that do that. To elaborate, Teller says, "An axperiment in the natural sciences determines a spatiotemporal fact, which then may or may not be generalized beyond the local place and time to which it directly applies" (Teller, 1980, 802). Thus, a correct mathematical proof establishes a non-empirically bound result that is also non-spatiotemporal. Given the evidence thus far, I don't see how this doesn't extend to CAPs.

Conclusion

After presenting the concerns that Tymoczko and others had about the a priori status of CAPs and contrasting them to theories on extended cognition and the extended mind I have concluded that there is no reason for the results of CAPs to not be warranted by a priori means. The reasoning for this is that through evolution, our mind has evolved to offload internal capacities into the environment. Through this we form a cognitive symbiotic relationships with non-organic entities in order to complete certain tasks that would be to complex to complete if they were solely localized inside of the skull. Thus, when we use cognitive aids, such as computers, to complete theorems that are way too tedious to solve in a lifetime, we are just extending our mind and methodology of proof-checking into the computer. Thus, the a priori justification is preserved.

Lastly, this paper has helped me appreciate the interface that philosophy of min and philosophy of mathematics have on each other. In using principles and theories that I once thought were solely contained inside the domain of philosophy of mind I have been able to adequately defend problems that arise in both philosophy of mathematics and epistemology. I believe this shows a connection amongst the field and hopefully philosophers in the future will continue cross boundaries to aid other sub-disciplines of philosophy.

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